MMATH II- TOPOLOGY IV - SEMESTRAL EXAM.

Max. marks : 60 Time : 3 hours

Answer all questions. You may use results proved in class after correctly stating them. Any other claim must be accompanied by a proof.

(1) Show for an open set $U \subseteq \mathbb{R}^2$ there is a commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \Omega^0(U) & \xrightarrow{d} & \Omega^1(U) & \xrightarrow{d} & \Omega^2(U) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle f_1} & & \downarrow{\scriptstyle f_2} & & \downarrow{\scriptstyle f_3} & & \\ 0 & \longrightarrow & C^\infty(U,\mathbb{R}) & \xrightarrow{\text{grad}} & C^\infty(U,\mathbb{R}^2) & \xrightarrow{\text{rot}} & C^\infty(U,\mathbb{R}) & \longrightarrow & 0 \end{array}$$

with f_1, f_2, f_3 isomorphisms. Recall that $\text{rot}(\phi_1, \phi_2) = (\partial\phi_1/\partial x_2) - (\partial\phi_2/\partial x_1)$. [10]

(2) Let $U \subseteq \mathbb{R}^n$ be an open set and $f : U \longrightarrow \mathbb{R}^n$ an injective continuous map. Show that $f(U)$ is open in $\mathbb{R}^n$ and f maps U homeomorphically onto $f(U)$. [10]

(3) State the Jordan-Brouwer separation theorem. Let $\Sigma \subseteq \mathbb{R}^n$ $(n \geq 2)$ be homeomorphic to S^{n-1} and let U_1 and U_2 respectively be the interior and exterior domains of Σ. Compute $H^p(U_1)$ and $H^p(U_2)$ for $p \geq 0$. [2+8]

(4) Construct a volume form on S^{n-1}, $n \geq 2$. [7]

(5) Show that $\mathbb{R}P^{n-1}$ is orientable if and only if n is even. [8]

(6) Compute $H^p(\mathbb{R}P^{n-1})$, $n \geq 2$, $p \geq 0$. [15]